| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| | | **Important Instructions to the Examiners:**<br>1) The Answers should be examined by key words and not as word-to-word as given in the model answer scheme.<br>2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.<br>3) The language errors such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and Communication Skills.)<br>4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by the candidate and those in the model answer may vary. The examiner may give credit for any equivalent figure drawn.<br>5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's Answers and the model answer.<br>6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidate's understanding.<br>7) For programming language papers, credit may be given to any other program based on equivalent concept. | | |

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|---|---|---|---|---|
| **1)** | | **Attempt any TEN of the following:** | | |
| | a) | If $f(x)=\cos x$, show that $f(3x)=4f^3(x)-3f(x)$ | | |
| | Ans. | $f(3x)=\cos(3x)$ | ½ | |
| | | $=4\cos^3 x-3\cos x$ | 1 | |
| | | $=4f^3(x)-3f(x)$ | ½ | **2** |
| | | **OR** | | |
| | | $LHS = f(3x)$ | | |
| | | $=\cos(3x)$ | ½ | |
| | | $=4\cos^3 x-3\cos x$ | 1 | |
| | | $RHS=4f^3(x)-3f(x)$ | | |
| | | $=4\cos^3 x-3\cos x$ | ½ | |
| | | $\therefore LHS = RHS$ | | **2** |
| | | ------------------------------------------------------------------------------------- | | |
| | b) | Express in the form of $a+ib$, $\frac{1+i}{2-i}$, where $a, b\in R,\ i=\sqrt{-1}$ | | |
| | Ans. | | | |
| | | $\frac{1+i}{2-i}=\frac{1+i}{2-i}\times\frac{2+i}{2+i}$ | ½ | |
| | | $=\frac{2+i+2i+i^2}{2^2-i^2}$ | | |
| | | $=\frac{2+3i-1}{4-(-1)}$ | 1 | |
| | | $=\frac{1+3i}{5}$ *or* $\frac{1}{5}+\frac{3}{5}i$ | ½ | **2** |
| | | **OR** | | |
| | | $\frac{1+i}{2-i}=\frac{1+i}{2-i}\times\frac{2+i}{2+i}$ | ½ | |
| | | $=\frac{2+i+2i-1}{2^2-i^2}$ | 1 | |
| | | $=\frac{1+3i}{5}$ *or* $\frac{1}{5}+\frac{3}{5}i$ | ½ | **2** |

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|---|---|---|---|---|
| **1)** | | **OR** | | |
| | | $Let\ a+ib=\frac{1+i}{2-i}$ | | |
| | | $\therefore (a+ib)(2-i)=1+i$ | | |
| | | $\therefore 2a-ai+2bi-bi^2=1+i$ | | |
| | | $\therefore 2a-ai+2bi+b=1+i$ | | |
| | | $\therefore (2a+b)+(-a+2b)i=1+i$ | ½ | |
| | | $\therefore 2a+b=1 \quad and \quad -a+2b=1$ | | |
| | | $\therefore a=\frac{1}{5} \quad and \quad b=\frac{3}{5}$ | ½ + ½ | |
| | | $\therefore \frac{1+i}{2-i}=\frac{1}{5}+\frac{3}{5}i$ | ½ | **2** |
| | | ------------------------------------------------------------------------------------ | | |
| | c) | Evaluate $\lim\limits_{x\to 0}\frac{1}{\sqrt{x+1}-1}$ | | |
| | Ans. | $\lim\limits_{x\to 0}\frac{1}{\sqrt{x+1}-1}=\frac{1}{\sqrt{0+1}-1}$ | 1 | |
| | | $=\infty$ | 1 | |
| | | **OR** | | **2** |
| | | $\lim\limits_{x\to 0}\frac{1}{\sqrt{x+1}-1}=\lim\limits_{x\to 0}\frac{1}{\sqrt{x+1}-1}\times\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}$ | | |
| | | $=\lim\limits_{x\to 0}\frac{\sqrt{x+1}+1}{x}$ | | |
| | | $=\frac{\sqrt{0+1}+1}{0}$ | 1 | |
| | | $=\infty$ | 1 | **2** |
| | | ------------------------------------------------------------------------------------ | | |
| | d) | Evaluate $\lim\limits_{x\to 0}\frac{2^x-1}{\sin 2x}$ | | |
| | Ans. | $\lim\limits_{x\to 0}\frac{2^x-1}{\sin 2x}=\lim\limits_{x\to 0}\frac{2^x-1}{x}\times\frac{x}{\sin 2x}$ | | |
| | | $=\lim\limits_{x\to 0}\frac{2^x-1}{x}\times\frac{2x}{\sin 2x}\times\frac{1}{2}$ | ½ | |
| | | $=\log 2\times 1\times\frac{1}{2}$ | 1 | |
| | | $=\frac{1}{2}\log 2$ | ½ | **2** |

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|---|---|---|---|---|
| **1)** | | **OR** | | |
| | | $\lim_{x\to 0}\frac{2^x-1}{\sin 2x}=\lim_{x\to 0}\frac{\frac{2^x-1}{x}}{\frac{\sin 2x}{x}}$ | | |
| | | $=\lim_{x\to 0}\frac{\frac{2^x-1}{x}}{\frac{\sin 2x}{2x}\times 2}$ | ½ | |
| | | $=\frac{\log 2}{1\times 2}$ | 1 | |
| | | $=\frac{\log 2}{2}$ | ½ | **2** |
| | e) | If $f(x)=3x^2-5x+7$, show that $f(-1)=3f(1)$ | | |
| | Ans. | $\therefore f(-1)=3(-1)^2-5(-1)+7$ | | |
| | | $=15$ | ½ | |
| | | $f(1)=3(1)^2-5(1)+7$ | ½ | |
| | | $=5$ | ½ | |
| | | $\therefore f(-1)=3f(1)$ | ½ | **2** |
| | f) | Find x and y, if $x(1-i)+y(2+i)+6=0$ | | |
| | Ans. | $x(1-i)+y(2+i)+6=0$ | | |
| | | $\therefore x-xi+2y+yi+6=0$ | ½ | |
| | | $\therefore (x+2y+6)+(-x+y)i=0$ | ½ | |
| | | $\therefore x+2y+6=0 \quad and \quad -x+y=0$ | | |
| | | $\therefore x=-2$ | ½ | |
| | | $y=-2$ | ½ | **2** |
| | g) | Evaluate $\lim_{x\to 0}\frac{1-\cos 2x}{x^2}$ | | |
| | Ans. | $\lim_{x\to 0}\frac{1-\cos 2x}{x^2}=\lim_{x\to 0}\frac{2\sin^2 x}{x^2}$ | ½ | |
| | | $=\lim_{x\to 0}2\left(\frac{\sin x}{x}\right)^2$ | ½ | |
| | | $=2(1)^2$ | ½ | |
| | | $=2$ | ½ | **2** |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| **1)** | h) | If $y=\cos^{-1}(\sin x)$, find $\frac{dy}{dx}$. | | |
| | Ans. | $y=\cos^{-1}(\sin x)$ | | |
| | | $\therefore \frac{dy}{dx}=-\frac{1}{\sqrt{1-\sin^2 x}}\cdot\frac{d}{dx}(\sin x)$ | ½ | |
| | | $=-\frac{1}{\sqrt{1-\sin^2 x}}\cdot\cos x$ | ½ | |
| | | $=-\frac{1}{\sqrt{\cos^2 x}}\cdot\cos x$ | | |
| | | $=-\frac{1}{\cos x}\cdot\cos x$ | ½ | |
| | | $=-1$ | ½ | **2** |
| | | **OR** | | |
| | | $y=\cos^{-1}(\sin x)$ | | |
| | | $=\cos^{-1}\left[\cos\left(\frac{\pi}{2}-x\right)\right]$ | ½ | |
| | | $=\frac{\pi}{2}-x$ | ½ | |
| | | $\therefore \frac{dy}{dx}=0-1$ | ½ | |
| | | $=-1$ | ½ | **2** |
| | i) | If $y=e^x\cdot\sin x\cdot\cos x$, find $\frac{dy}{dx}$ | | |
| | Ans. | $\therefore \frac{dy}{dx}=e^x\cdot\sin x\cdot\frac{d}{dx}(\cos x)+e^x\cdot\cos x\frac{d}{dx}(\sin x)+\sin x\cdot\cos x\frac{d}{dx}(e^x)$ | 1 | |
| | | $=e^x\cdot\sin x\cdot(-\sin x)+e^x\cdot\cos x\cdot\cos x+\sin x\cdot\cos x\cdot e^x$ | | |
| | | $=-e^x\cdot\sin^2 x+e^x\cdot\cos^2 x+e^x\cdot\sin x\cdot\cos x$ | 1 | **2** |
| | | $=e^x\cdot\left(-\sin^2 x+\cos^2 x+\sin x\cdot\cos x\right)$ | | |
| | | **OR** | | |
| | | $y=e^x\cdot\sin x\cdot\cos x=\frac{1}{2}\cdot e^x\cdot\sin 2x$ | | |
| | | $\therefore \frac{dy}{dx}=\frac{1}{2}\left[e^x\cdot\frac{d}{dx}(\sin 2x)+\sin 2x\cdot\frac{d}{dx}(e^x)\right]$ | 1 | |
| | | $=\frac{1}{2}\left[e^x\cdot\cos 2x\cdot 2+\sin 2x\cdot e^x\right]$ | 1 | |
| | | $=\frac{1}{2}e^x\cdot(2\cos 2x+\sin 2x)$ | | **2** |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| **1)** | | | | |
| | j) | Find $\frac{dy}{dx}$, if $y=x^x$ | | |
| | Ans. | $\therefore \log y = x\cdot\log x$ | ½ | |
| | | $\therefore \frac{1}{y}\cdot\frac{dy}{dx} = x\cdot\frac{1}{x}+\log x\cdot 1$ | ½+½ | |
| | | $\therefore \frac{dy}{dx}=y\left(1+\log x\right)$ | ½ | **2** |
| | | ------------------------------------------------------------------------------------ | | |
| | k) | Find first two real roots of equation $x^3-2x-5=0$ using bisection method. | | |
| | Ans. | $f\left(x\right)=x^3-2x-5$ | | |
| | | $\therefore f\left(2\right)=-1 \qquad ---(*)$ | ½ | |
| | | $f\left(3\right)=16$ | ½ | |
| | | $\therefore$ the root is in $\left(2,\ 3\right)$ | | |
| | | $\therefore x_1=\frac{2+3}{2}=2.5$ | ½ | |
| | | $\therefore f\left(2.5\right)=5.625$ | | |
| | | $\therefore$ the root is in $\left(2,\ 2.5\right)$ | | |
| | | $\therefore x_2=\frac{2+2.5}{2}=2.25$ | ½ | **2** |
| | | **OR** | | |
| | | $f\left(x\right)=x^3-2x-5$ | | |
| | | $\therefore f\left(2\right)=-1$ | ½ | |
| | | $f\left(3\right)=16$ | ½ | |
| | | $\therefore$ the root is in $\left(2,\ 3\right)$ | | |

| a | b | $x=\frac{a+b}{2}$ | $f\left(x\right)$ |
|---|---|---|---|
| 2 | 3 | 2.5 | 5.625 |
| 2 | 2.5 | 2.25 | --- |

| | | | | |
|---|---|---|---|---|
| | | | ½ | |
| | | | ½ | **2** |
| | | **Note (*) :** In **numerical methods problems only**, writing directly the exact values of functions, such as here in this example f(2) or f(3), is allowed. | | |

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| **1)** | *l*) | Find the first iteration by using Jacobi's method for the following system of equations:<br>$5x-y+z=10,\ \ x+2y=6,\ \ x+y+5z=-1$ | | |
| | Ans. | $5x-y+z=10$<br>$x+2y=6$<br>$x+y+5z=-1$<br>$\therefore x=\frac{1}{5}(10+y-z)$<br>$y=\frac{1}{2}(6-x)$<br>$z=\frac{1}{5}(-1-x-y)$ | 1 | |
| | | Start with $x_0=0,\ \ y_0=0,\ \ z_0=0$<br>$\therefore x_1=2$<br>$y_1=3$<br>$z_1=-0.2$ | 1 | **2** |
| | | ------------------------------------------------------------------------------------- | | |
| **2)** | | **Attempt any Four of the following:** | | |
| | a) | If $f(x)=\frac{x-4}{4x-1}$, then show that $f\left[f(x)\right]=x$. | | |
| | Ans. | $f(x)=\frac{x-4}{4x-1}$<br>$\therefore f\left[f(x)\right]=\frac{f(x)-4}{4f(x)-1}$ | 1 | |
| | | $=\frac{\frac{x-4}{4x-1}-4}{4\left(\frac{x-4}{4x-1}\right)-1}$ | 1 | |
| | | $=\frac{\frac{x-4-4(4x-1)}{4x-1}}{\frac{4(x-4)-1(4x-1)}{4x-1}}$<br>$=\frac{x-4-16x+4}{4x-16-4x+1}$ | 1 | |
| | | $=\frac{-15x}{-15}$ | ½ | |
| | | $=x$ | ½ | **4** |

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|---|---|---|---|---|
| **2)** | b) | If $f(x)=\log\left[\frac{1+x}{1-x}\right]$, then show that $f(a)+f(b)=f\left[\frac{a+b}{1+ab}\right]$ | | |
| | Ans. | $\therefore f(a)=\log\left[\frac{1+a}{1-a}\right]$ | | |
| | | $f(b)=\log\left[\frac{1+b}{1-b}\right]$ | | |
| | | $\therefore LHS=f(a)+f(b)$ | | |
| | | $=\log\left[\frac{1+a}{1-a}\right]+\log\left[\frac{1+b}{1-b}\right]$ | ½+½ | |
| | | $=\log\left[\frac{1+a}{1-a}\times\frac{1+b}{1-b}\right]$ | ½ | |
| | | $=\log\left[\frac{1+a+b+ab}{1-a-b+ab}\right]$ | ½ | |
| | | $RHS=f\left[\frac{a+b}{1+ab}\right]$ | | |
| | | $=\log\left[\frac{1+\frac{a+b}{1+ab}}{1-\frac{a+b}{1+ab}}\right]$ | ½ | |
| | | $=\log\left[\frac{\frac{1+ab+(a+b)}{1+ab}}{\frac{1+ab-(a+b)}{1+ab}}\right]$ | ½ | |
| | | $=\log\left[\frac{1+ab+a+b}{1+ab-a-b}\right]$ | ½ | |
| | | $\therefore LHS=RHS$ | ½ | **4** |
| | | ------------------------------------------------------------------------------------ | | |
| | c) | Using Euler's formulae prove that $\cos 2\theta=\cos^2\theta-\sin^2\theta$ | | |
| | Ans. | $\cos 2\theta=\frac{e^{2i\theta}+e^{-2i\theta}}{2}$ | ½ | |
| | | $\cos^2\theta-\sin^2\theta=\left(\frac{e^{i\theta}+e^{-i\theta}}{2}\right)^2-\left(\frac{e^{i\theta}-e^{-i\theta}}{2i}\right)^2$ | ½ | |
| | | $=\frac{\left(e^{i\theta}\right)^2+2e^{i\theta}e^{-i\theta}+\left(e^{-i\theta}\right)^2}{4}-\frac{\left(e^{i\theta}\right)^2-2e^{i\theta}e^{-i\theta}+\left(e^{-i\theta}\right)^2}{-4}$ | 1 | |
| | | $=\frac{e^{2i\theta}+2+e^{-2i\theta}}{4}-\frac{e^{2i\theta}-2+e^{-2i\theta}}{-4}$ | ½ | |

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| **2)** | | $=\frac{e^{2i\theta}+2+e^{-2i\theta}}{4}+\frac{e^{2i\theta}-2+e^{-2i\theta}}{4}$ | | |
| | | $=\frac{e^{2i\theta}+2+e^{-2i\theta}+e^{2i\theta}-2+e^{-2i\theta}}{4}$ | ½ | |
| | | $=\frac{2e^{2i\theta}+2e^{-2i\theta}}{4}$ | ½ | |
| | | $=\frac{e^{2i\theta}+e^{-2i\theta}}{2}$ | | |
| | | $=\cos 2\theta$ | ½ | **4** |
| | | ------------------------------------------------------------------------------------ | | |
| | d) | Simplify using DeMoivre's theorem: | | |
| | | $\frac{(\cos 5\theta - i\sin 5\theta)^{\frac{2}{5}}\left(\cos\frac{2}{7}\theta + i\sin\frac{2}{7}\theta\right)^{2}}{(\cos 4\theta + i\sin 4\theta)^{\frac{1}{4}}\left(\cos\frac{2}{3}\theta - i\sin\frac{2}{3}\theta\right)^{3}}$ | | |
| | Ans. | $\frac{(\cos 5\theta - i\sin 5\theta)^{\frac{2}{5}}\left(\cos\frac{2}{7}\theta + i\sin\frac{2}{7}\theta\right)^{2}}{(\cos 4\theta + i\sin 4\theta)^{\frac{1}{4}}\left(\cos\frac{2}{3}\theta - i\sin\frac{2}{3}\theta\right)^{3}}$ | | |
| | | $=\frac{(\cos\theta + i\sin\theta)^{-5\times\frac{2}{5}}(\cos\theta + i\sin\theta)^{\frac{2}{7}\times 2}}{(\cos\theta + i\sin\theta)^{4\times\frac{1}{4}}(\cos\theta + i\sin\theta)^{-\frac{2}{3}\times 3}}$ | ½+½+<br>½+½ | |
| | | $=\frac{(\cos\theta + i\sin\theta)^{-2}(\cos\theta + i\sin\theta)^{\frac{4}{7}}}{(\cos\theta + i\sin\theta)^{1}(\cos\theta + i\sin\theta)^{-2}}$ | 1 | |
| | | $=(\cos\theta + i\sin\theta)^{-2+\frac{4}{7}-1+2}$ | | |
| | | $=(\cos\theta + i\sin\theta)^{-\frac{3}{7}}$ | ½ | |
| | | $=\cos\frac{3}{7}\theta - i\sin\frac{3}{7}\theta$ | ½ | **4** |
| | | **OR** | | |
| | | $(\cos 5\theta - i\sin 5\theta)^{\frac{2}{5}}=(\cos\theta + i\sin\theta)^{-5\times\frac{2}{5}}=(\cos\theta + i\sin\theta)^{-2}$ | ½ | |
| | | $\left(\cos\frac{2}{7}\theta + i\sin\frac{2}{7}\theta\right)^{2}=(\cos\theta + i\sin\theta)^{\frac{2}{7}\times 2}=(\cos\theta + i\sin\theta)^{\frac{4}{7}}$ | ½ | |
| | | $(\cos 4\theta + i\sin 4\theta)^{\frac{1}{4}}=(\cos\theta + i\sin\theta)^{4\times\frac{1}{4}}=(\cos\theta + i\sin\theta)^{1}$ | ½ | |
| | | $\left(\cos\frac{2}{3}\theta - i\sin\frac{2}{3}\theta\right)^{3}=(\cos\theta + i\sin\theta)^{-\frac{2}{3}\times 3}=(\cos\theta + i\sin\theta)^{-2}$ | ½ | |

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| **2)** | | $\therefore \dfrac{(\cos 5\theta - i\sin 5\theta)^{\frac{2}{5}}\left(\cos\frac{2}{7}\theta + i\sin\frac{2}{7}\theta\right)^{2}}{(\cos 4\theta + i\sin 4\theta)^{\frac{1}{4}}\left(\cos\frac{2}{3}\theta - i\sin\frac{2}{3}\theta\right)^{3}}$ | | |
| | | $= \dfrac{(\cos\theta + i\sin\theta)^{-2}(\cos\theta + i\sin\theta)^{\frac{4}{7}}}{(\cos\theta + i\sin\theta)^{1}(\cos\theta + i\sin\theta)^{-2}}$ | 1 | |
| | | $= (\cos\theta + i\sin\theta)^{-2+\frac{4}{7}-1+2}$ | | |
| | | $= (\cos\theta + i\sin\theta)^{-\frac{3}{7}}$ | ½ | |
| | | $= \cos\frac{3}{7}\theta - i\sin\frac{3}{7}\theta$ | ½ | **4** |
| | | ------------------------------------------------------------------------------------- | | |
| | e) | Find the cube roots of unity. | | |
| | Ans. | Let $z = 1 = 1 + 0i$ $\quad a = 1,\ b = 0$ | | |
| | | $\therefore r = \sqrt{1^2 + 0^2} = 1$ | ½ | |
| | | $\theta = \tan^{-1}\left(\frac{0}{1}\right) = 0$ | ½ | |
| | | $\therefore z = 1 = r(\cos\theta + i\sin\theta)$ | | |
| | | $= \cos 0 + i\sin 0$ | ½ | |
| | | $= \cos 2\pi k + i\sin 2\pi k$ | ½ | |
| | | $\therefore 1^{1/3} = [\cos 2\pi k + i\sin 2\pi k]^{1/3}$ | | |
| | | $= \cos\left(\frac{2\pi k}{3}\right) + i\sin\left(\frac{2\pi k}{3}\right)$ | ½ | |
| | | $For\ k = 0, \quad 1^{1/3} = \cos(0) + i\sin(0)$ | | |
| | | $= 1 + 0 = 1$ | ½ | |
| | | $For\ k = 1, \quad 1^{1/3} = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)$ | | |
| | | $= -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ | ½ | |
| | | $For\ k = 2, \quad 1^{1/3} = \cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right)$ | | |
| | | $= -\frac{1}{2} - \frac{\sqrt{3}}{2}i$ | ½ | **4** |

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| **2)** | | | | |
| | f) | Simplify $1+i^{100}+i^{10}+i^{50}$ | | |
| | Ans. | $1+i^{100}+i^{10}+i^{50}$ | | |
| | | $=1+\left(i^4\right)^{25}+\left(i^2\right)^5+\left(i^2\right)^{25}$ | 1 | |
| | | $=1+(1)^{25}+(-1)^5+(-1)^{25}$ | 1 | |
| | | $=1+1-1-1$ | 1 | |
| | | $=0$ | 1 | **4** |
| | | **OR** | | |
| | | $i^{100}=\left(i^4\right)^{25}=(1)^{25}=1$ | 1 | |
| | | $i^{10}=\left(i^2\right)^5=(-1)^5=-1$ | 1 | |
| | | $i^{50}=\left(i^2\right)^{25}=(-1)^{25}=-1$ | 1 | |
| | | $\therefore 1+i^{100}+i^{10}+i^{50}=1+1-1-1$ | | **4** |
| | | $=0$ | 1 | |
| | | ---------------------------------------------------------------------------------- | | |
| **3)** | | **Attempt any Four of the followings:** | | |
| | a) | If $f(x)=ax^2+bx+3$ and $f(1)=4,\ f(2)=11$, find 'a' and 'b'. | | |
| | Ans. | $f(x)=ax^2+bx+3$ | | |
| | | $\therefore f(1)=a(1)^2+b(1)+3$ | | |
| | | $=a+b+3$ | 1 | |
| | | $f(2)=a(2)^2+b(2)+3$ | | |
| | | $=4a+2b+3$ | 1 | |
| | | $But\ f(1)=4,\ f(2)=11$ | | |
| | | $\therefore a+b+3=4$ | | |
| | | $4a+2b+3=11$ | | |
| | | $\therefore a+b=1$ | ½ | |
| | | $4a+2b=8$ | ½ | |
| | | $\therefore a=3$ | ½ | |
| | | $b=-2$ | ½ | **4** |
| | | **OR** | | |

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| **3)** | | $f(1)=4$ | | |
| | | $\therefore a(1)^2+b(1)+3=4$ | 1 | |
| | | $\therefore a+b+3=4$ | | |
| | | $\therefore a+b=1 \qquad ---(i)$ | ½ | |
| | | $f(2)=11$ | | |
| | | $a(2)^2+b(2)+3=11$ | | |
| | | $\therefore 4a+2b+3=11$ | 1 | |
| | | $\therefore 4a+2b=8 \qquad ---(ii)$ | ½ | |
| | | $\therefore$ by $(i)$ and $(ii)$, | | |
| | | $a=3$ | ½ | |
| | | $b=-2$ | ½ | **4** |
| | | ------------------------------------------------------------------------------------------ | | |
| | b) | If $f(x)=\sin x, \quad g(x)=\cos x$, prove that | | |
| | | i) $f(x+y)=f(x)g(y)+g(x)f(y)$ | | |
| | | ii) $g(m-n)=g(m)g(n)+f(m)f(n)$ | | |
| | Ans. | $f(x)=\sin x, \quad g(x)=\cos x$ | | |
| | | $i) f(x+y)=\sin(x+y)$ | ½ | |
| | | $=\sin x\cos y+\cos x\sin y$ | ½ | |
| | | $=f(x)g(y)+g(x)f(y)$ | 1 | |
| | | $ii) g(m-n)=\cos(m-n)$ | ½ | |
| | | $=\cos m\cos n+\sin m\sin n$ | ½ | |
| | | $=g(m)g(n)+f(m)f(n)$ | 1 | **4** |
| | | ------------------------------------------------------------------------------------------ | | |
| | c) | Evaluate $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin^2 x-\cos^2 x}{1-\tan x}$ | | |
| | Ans. | $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin^2 x-\cos^2 x}{1-\tan x}=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{(\sin x-\cos x)(\sin x+\cos x)}{1-\dfrac{\sin x}{\cos x}}$ | 1 | |
| | | $=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{(\sin x-\cos x)(\sin x+\cos x)}{\dfrac{\cos x-\sin x}{\cos x}}$ | | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| **3)** | | $=\lim\limits_{x\to\frac{\pi}{4}}\frac{(\sin x-\cos x)(\sin x+\cos x)}{-(\sin x-\cos x)}\times\cos x$ | | |
| | | $=\lim\limits_{x\to\frac{\pi}{4}}\frac{\sin x+\cos x}{-1}\times\cos x$ | 1 | |
| | | $=\frac{\sin\frac{\pi}{4}+\cos\frac{\pi}{4}}{-1}\times\cos\frac{\pi}{4}$ | 1 | |
| | | $=-1$ | 1 | **4** |
| | | **OR** | | |
| | | $\lim\limits_{x\to\frac{\pi}{4}}\frac{\sin^2 x-\cos^2 x}{1-\tan x}=\lim\limits_{x\to\frac{\pi}{4}}\frac{\sin^2 x-\cos^2 x}{1-\tan x}\times\frac{1+\tan x}{1+\tan x}$ | | |
| | | $=\lim\limits_{x\to\frac{\pi}{4}}\frac{\sin^2 x-\cos^2 x}{1-\tan^2 x}\times(1+\tan x)$ | | |
| | | $=\lim\limits_{x\to\frac{\pi}{4}}\frac{\sin^2 x-\cos^2 x}{1-\frac{\sin^2 x}{\cos^2 x}}\times(1+\tan x)$ | | |
| | | $=\lim\limits_{x\to\frac{\pi}{4}}\frac{-(\cos^2 x-\sin^2 x)}{\frac{\cos^2 x-\sin^2 x}{\cos^2 x}}\times(1+\tan x)$ | 1 | |
| | | $=-\lim\limits_{x\to\frac{\pi}{4}}\cos^2 x(1+\tan x)$ | 1 | |
| | | $=-\cos^2\frac{\pi}{4}\left(1+\tan\frac{\pi}{4}\right)$ | 1 | |
| | | $=-1$ | 1 | **4** |
| | | **OR** | | |
| | | $\lim\limits_{x\to\frac{\pi}{4}}\frac{\sin^2 x-\cos^2 x}{1-\tan x}=\lim\limits_{x\to\frac{\pi}{4}}\frac{-\cos 2x}{1-\tan x}$ | | |
| | | $=-\lim\limits_{x\to\frac{\pi}{4}}\frac{\left.1-\tan^2 x\middle/1+\tan^2 x\right.}{1-\tan x}$ | | |
| | | $=-\lim\limits_{x\to\frac{\pi}{4}}\frac{(1-\tan x)(1+\tan x)}{1-\tan x}\times\frac{1}{1+\tan^2 x}$ | 1 | |
| | | $=-\lim\limits_{x\to\frac{\pi}{4}}(1+\tan x)\times\frac{1}{1+\tan^2 x}$ | 1 | |
| | | $=-\left(1+\tan\frac{\pi}{4}\right)\times\frac{1}{1+\tan^2\frac{\pi}{4}}$ | 1 | |
| | | $=-1$ | 1 | **4** |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| **3)** | d) | Evaluate $\lim\limits_{x\to\infty} x\left[\sqrt{x^2+1}-\sqrt{x^2-1}\right]$ | | |
| | Ans. | $\lim\limits_{x\to\infty} x\left[\sqrt{x^2+1}-\sqrt{x^2-1}\right]=\lim\limits_{x\to\infty} x\left[\sqrt{x^2+1}-\sqrt{x^2-1}\right]\times\frac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^2+1}+\sqrt{x^2-1}}$ | ½ | |
| | | $=\lim\limits_{x\to\infty} x\times\frac{\left(x^2+1\right)-\left(x^2-1\right)}{\sqrt{x^2+1}+\sqrt{x^2-1}}$ | | |
| | | $=\lim\limits_{x\to\infty} x\times\frac{x^2+1-x^2+1}{\sqrt{x^2+1}+\sqrt{x^2-1}}$ | | |
| | | $=\lim\limits_{x\to\infty}\frac{2x}{\sqrt{x^2+1}+\sqrt{x^2-1}}$ | 1 | |
| | | $=\lim\limits_{x\to\infty}\frac{\frac{2x}{x}}{\frac{\sqrt{x^2+1}+\sqrt{x^2-1}}{x}}$ | ½ | |
| | | $=\lim\limits_{x\to\infty}\frac{2}{\sqrt{1+\frac{1}{x^2}}+\sqrt{1-\frac{1}{x^2}}}$ | 1 | |
| | | $=\frac{2}{\sqrt{1+0}+\sqrt{1-0}}$ | ½ | |
| | | $=1$ | ½ | **4** |
| | | **OR** | | |
| | | $Put\quad x=\frac{1}{t}\qquad\therefore t\to 0$ | | |
| | | $\lim\limits_{x\to\infty} x\left[\sqrt{x^2+1}-\sqrt{x^2-1}\right]=\lim\limits_{t\to 0}\frac{1}{t}\left[\sqrt{\frac{1}{t^2}+1}-\sqrt{\frac{1}{t^2}-1}\right]$ | | |
| | | $=\lim\limits_{t\to 0}\frac{1}{t}\left[\sqrt{\frac{1+t^2}{t^2}}-\sqrt{\frac{1-t^2}{t^2}}\right]$ | | |
| | | $=\lim\limits_{t\to 0}\frac{1}{t}\left[\frac{\sqrt{1+t^2}}{t}-\frac{\sqrt{1-t^2}}{t}\right]$ | | |
| | | $=\lim\limits_{t\to 0}\frac{\sqrt{1+t^2}-\sqrt{1-t^2}}{t^2}$ | 1 | |
| | | $=\lim\limits_{t\to 0}\frac{\sqrt{1+t^2}-\sqrt{1-t^2}}{t^2}\times\frac{\sqrt{1+t^2}+\sqrt{1-t^2}}{\sqrt{1+t^2}+\sqrt{1-t^2}}$ | ½ | |
| | | $=\lim\limits_{t\to 0}\frac{\left(1+t^2\right)-\left(1-t^2\right)}{t^2}\times\frac{1}{\sqrt{1+t^2}+\sqrt{1-t^2}}$ | | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| **3)** | | $=\lim_{t\to 0}\frac{2t^2}{t^2}\times\frac{1}{\sqrt{1+t^2}+\sqrt{1-t^2}}$ | ½ | |
| | | $=\lim_{t\to 0}\frac{2}{\sqrt{1+t^2}+\sqrt{1-t^2}}$ | 1 | |
| | | $=\frac{2}{\sqrt{1+0}+\sqrt{1-0}}$ | ½ | |
| | | $=1$ | ½ | **4** |
| | | ------------------------------------------------------------------------------------- | | |
| | e) | Evaluate $\lim_{x\to 4}\frac{x^2-7x+12}{x^3-64}$ | | |
| | Ans. | $\lim_{x\to 4}\frac{x^2-7x+12}{x^3-64}=\lim_{x\to 4}\frac{(x-4)(x-3)}{(x-4)(x^2+4x+16)}$ | 1 | |
| | | $=\lim_{x\to 4}\frac{x-3}{x^2+4x+16}$ | 1 | |
| | | $=\frac{4-3}{(4)^2+4(4)+16}$ | 1 | |
| | | $=\frac{1}{48}$ | 1 | **4** |
| | | ------------------------------------------------------------------------------------- | | |
| | f) | Evaluate $\lim_{x\to 0}\frac{a^x+a^{-x}-2}{\sin^2 x}$ | | |
| | Ans. | $\lim_{x\to 0}\frac{a^x+a^{-x}-2}{\sin^2 x}=\lim_{x\to 0}\frac{a^x+\frac{1}{a^x}-2}{\sin^2 x}$ | | |
| | | $=\lim_{x\to 0}\frac{\frac{(a^x)^2+1-2a^x}{a^x}}{\sin^2 x}$ | | |
| | | $=\lim_{x\to 0}\frac{(a^x-1)^2}{x^2}\times\frac{x^2}{\sin^2 x}\times\frac{1}{a^x}$ | 1 | |
| | | $=\lim_{x\to 0}\left(\frac{a^x-1}{x}\right)^2\times\left(\frac{x}{\sin x}\right)^2\times\frac{1}{a^x}$ | 1 | |
| | | $=(\log a)^2\times 1\times\frac{1}{a^0}$ | 1 | |
| | | $=(\log a)^2$ | 1 | **4** |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| **4)** | | **Attempt any Four of the followings:** | | |
| | a) | If u and v are differential functions of x and $y=\frac{u}{v}$, prove that $\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ | | |
| | Ans. | Let $\delta x$ be infinitesimal increment in x and $\delta y$, $\delta u$, $\delta v$ be corresponding infinitesimal increments in y, u, v. | | |
| | | $\therefore y+\delta y=\frac{u+\delta u}{v+\delta v}$ | 1 | |
| | | $\therefore \delta y=\frac{u+\delta u}{v+\delta v}-y$<br>$=\frac{u+\delta u}{v+\delta v}-\frac{u}{v}$<br>$=\frac{uv+v\delta u-uv-u\delta v}{v^2+v\delta v}$<br>$=\frac{v\delta u-u\delta v}{v^2+v\delta v}$ | 1 | |
| | | $\therefore \frac{\delta y}{\delta x}=\frac{1}{\delta x}\left(\frac{v\delta u-u\delta v}{v^2+v\delta v}\right)=\frac{v\frac{\delta u}{\delta x}-u\frac{\delta v}{\delta x}}{v^2+v\delta v}$<br>$\therefore \lim_{\delta x\to 0}\frac{\delta y}{\delta x}=\lim_{\delta x\to 0}\left[\frac{v\frac{\delta u}{\delta x}-u\frac{\delta v}{\delta x}}{v^2+v\delta v}\right]$ | 1 | |
| | | $\therefore \lim_{\delta x\to 0}\frac{\delta y}{\delta x}=\frac{v\lim_{\delta x\to 0}\frac{\delta u}{\delta x}-u\lim_{\delta x\to 0}\frac{\delta v}{\delta x}}{\lim_{\delta x\to 0}\left(v^2+v\delta v\right)}$<br>$\therefore \frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ $\left(\text{As } \delta x\to 0 \text{ , } \delta v\to 0\right)$ | 1 | **4** |
| | | ---------------------------------------------------------------------------------------- | | |
| | b) | If $y=\sin^{-1}\left(3x-4x^3\right)$, find $\frac{dy}{dx}$. | | |
| | Ans. | $Put\ x=\sin\theta$<br>$\therefore y=\sin^{-1}\left(3x-4x^3\right)$<br>$=\sin^{-1}\left(3\sin\theta-4\sin^3\theta\right)$<br>$=\sin^{-1}\left(\sin 3\theta\right)$ | 1 | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| **4)** | | $=3\theta$ | 1 | |
| | | $=3\sin^{-1}x$ | 1 | |
| | | $\therefore \frac{dy}{dx}=3\cdot\frac{1}{\sqrt{1-x^2}}$ | 1 | **4** |
| | | **OR** | | |
| | | $y=\sin^{-1}\left(3x-4x^3\right)$ | | |
| | | $\therefore \frac{dy}{dx}=\frac{1}{\sqrt{1-\left(3x-4x^3\right)^2}}\times\frac{d}{dx}\left(3x-4x^3\right)$ | 1 | |
| | | $=\frac{1}{\sqrt{1-\left(3x-4x^3\right)^2}}\times\left(3-12x^2\right)$ | 1 | |
| | | $=\frac{1}{\sqrt{1-\left(9x^2-24x^4+16x^6\right)}}\times\left(3-12x^2\right)$ | | |
| | | $=\frac{1}{\sqrt{1-9x^2+24x^4-16x^6}}\times\left(3-12x^2\right)$ | | |
| | | $=\frac{1}{\sqrt{\left(1-x^2\right)\left(1-8x^2+16x^4\right)}}\times3\left(1-4x^2\right)$ | | |
| | | $=\frac{1}{\sqrt{\left(1-x^2\right)\left(1-4x^2\right)^2}}\times3\left(1-4x^2\right)$ | | |
| | | $=\frac{1}{\left(1-4x^2\right)\sqrt{1-x^2}}\times3\left(1-4x^2\right)$ | 1 | |
| | | $\therefore \frac{dy}{dx}=3\cdot\frac{1}{\sqrt{1-x^2}}$ | 1 | **4** |
| | | ------------------------------------------------------------------------------------------------ | | |
| | c) | Find $\frac{dy}{dx}$, if $13x^2+2x^2y+y^3=1$. | | |
| | Ans. | $13x^2+2x^2y+y^3=1$ | | |
| | | $\therefore 26x+2\left(x^2\frac{dy}{dx}+y\cdot2x\right)+3y^2\frac{dy}{dx}=0$ | 1 | |
| | | $\therefore 26x+2x^2\frac{dy}{dx}+4xy+3y^2\frac{dy}{dx}=0$ | 1 | |
| | | $\therefore 26x+4xy+\left(2x^2+3y^2\right)\frac{dy}{dx}=0$ | 1 | |
| | | $\therefore \frac{dy}{dx}=-\frac{26x+4xy}{2x^2+3y^2}$ | 1 | **4** |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| **4)** | | **OR** | | |
| | | $13x^2+2x^2y+y^3=1$ | | |
| | | $\therefore 26x+2\left(x^2\frac{dy}{dx}+y\cdot 2x\right)+3y^2\frac{dy}{dx}=0$ | 1 | |
| | | $\therefore 26x+2x^2\frac{dy}{dx}+4xy+3y^2\frac{dy}{dx}=0$ | 1 | |
| | | $\therefore \left(2x^2+3y^2\right)\frac{dy}{dx}=-26x-4xy$ | 1 | |
| | | $\therefore \frac{dy}{dx}=\frac{-26x-4xy}{2x^2+3y^2}$ | 1 | **4** |
| | | ------------------------------------------------------------------------------------ | | |
| | d) | Find the derivative of $(x)\sin^{-1}x$ | | |
| | Ans. | $Let\ y=(x)\sin^{-1}x$ | | |
| | | $\therefore \frac{dy}{dx}=x\cdot\frac{d}{dx}\left(\sin^{-1}x\right)+\sin^{-1}x\cdot\frac{d}{dx}(x)$ | 1 | |
| | | $=x\cdot\frac{1}{\sqrt{1-x^2}}+\sin^{-1}x\cdot 1$ | 1+1 | |
| | | $=\frac{x}{\sqrt{1-x^2}}+\sin^{-1}x$ | 1 | **4** |
| | | ------------------------------------------------------------------------------------ | | |
| | e) | Using first principle find the derivative of $f(x)=a^x$ | | |
| | Ans. | $f(x)=a^x$ | | |
| | | $\therefore f(x+h)=a^{x+h}$ | | |
| | | $\therefore \frac{dy}{dx}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ | | |
| | | $=\lim_{h\to 0}\frac{a^{x+h}-a^x}{h}$ | 1 | |
| | | $=\lim_{h\to 0}\frac{a^x\left(a^h-1\right)}{h}$ | 1 | |
| | | $=\lim_{h\to 0}a^x\left(\frac{a^h-1}{h}\right)$ | 1 | |
| | | $=a^x\log a$ | 1 | **4** |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| **4)** | | | | |
| | f) | Find $\frac{dy}{dx}$, if $y=\log\left(x+\sqrt{x^2+a^2}\right)$ | | |
| | Ans. | $\therefore \frac{dy}{dx}=\frac{1}{x+\sqrt{x^2+a^2}}\cdot\frac{d}{dx}\left(x+\sqrt{x^2+a^2}\right)$ | 1 | |
| | | $=\frac{1}{x+\sqrt{x^2+a^2}}\cdot\left(1+\frac{1}{2\sqrt{x^2+a^2}}\cdot\frac{d}{dx}\left(x^2+a^2\right)\right)$ | 1 | |
| | | $=\frac{1}{x+\sqrt{x^2+a^2}}\cdot\left(1+\frac{1}{2\sqrt{x^2+a^2}}\cdot 2x\right)$ | ½ | |
| | | $=\frac{1}{x+\sqrt{x^2+a^2}}\cdot\left(1+\frac{x}{\sqrt{x^2+a^2}}\right)$ | | |
| | | $=\frac{1}{\cancel{x+\sqrt{x^2+a^2}}}\cdot\left(\frac{\cancel{\sqrt{x^2+a^2}+x}}{\sqrt{x^2+a^2}}\right)$ | ½ | |
| | | $=\frac{1}{\sqrt{x^2+a^2}}$ | 1 | **4** |
| | | ----------------------------------------------------------------------------------- | | |
| **5)** | | **Attempt any Four of the followings:** | | |
| | a) | Evaluate $\lim\limits_{x\to 1}\frac{\sin \pi x}{x-1}$ | | |
| | Ans. | $Put\ \ x-1=t$ | | |
| | | $\therefore as\ x\to 1,\quad t\to 0$ | | |
| | | $\therefore \lim\limits_{x\to 1}\frac{\sin \pi x}{x-1}=\lim\limits_{t\to 0}\frac{\sin \pi\left(1+t\right)}{t}$ | 1 | |
| | | $=\lim\limits_{t\to 0}\frac{\sin\left(\pi+\pi t\right)}{t}$ | | |
| | | $=\lim\limits_{t\to 0}\frac{-\sin \pi t}{t}$ | 1 | |
| | | $=-\lim\limits_{t\to 0}\frac{\sin \pi t}{\pi t}\times\pi$ | 1 | |
| | | $=-1\times\pi$ | ½ | |
| | | $=-\pi$ | ½ | **4** |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| **5)** | | | | |
| | b) | Evaluate $\lim_{x\to0}\frac{x}{\sqrt{9-x+x^2}-3}$ | | |
| | Ans. | $\therefore \lim_{x\to0}\frac{x}{\sqrt{9-x+x^2}-3}=\lim_{x\to0}\frac{x}{\sqrt{9-x+x^2}-3}\times\frac{\sqrt{9-x+x^2}+3}{\sqrt{9-x+x^2}+3}$ | ½ | |
| | | $=\lim_{x\to0}\frac{x}{9-x+x^2-9}\times\left(\sqrt{9-x+x^2}+3\right)$ | ½ | |
| | | $=\lim_{x\to0}\frac{x}{x^2-x}\times\left(\sqrt{9-x+x^2}+3\right)$ | 1 | |
| | | $=\lim_{x\to0}\frac{x}{x(x-1)}\times\left(\sqrt{9-x+x^2}+3\right)$ | | |
| | | $=\lim_{x\to0}\frac{1}{x-1}\times\left(\sqrt{9-x+x^2}+3\right)$ | 1 | |
| | | $=\frac{1}{0-1}\times\left(\sqrt{9-0+0}+3\right)$ | ½ | |
| | | $=-6$ | ½ | **4** |
| | | ------------------------------------------------------------------------------------------ | | |
| | c) | Using bisection method find the approximate root of $x^2+x-3=0$ (carry out three iterations). | | |
| | Ans. | $x^2+x-3=0$ | | |
| | | $f(x)=x^2+x-3$ | | |
| | | $\therefore f(1)=-1$ | ½ | |
| | | $f(2)=3$ | ½ | |
| | | $\therefore$ the root is in $(1,\ 2)$. | ½ | |
| | | $\therefore x_1=\frac{1+2}{2}=1.5$ | ½ | |
| | | $\therefore f(1.5)=0.75$ | ½ | |
| | | $\therefore$ the root is in $(1,\ 1.5)$. | | |
| | | $\therefore x_2=\frac{1+1.5}{2}=1.25$ | ½ | |
| | | $\therefore f(1.25)=-0.188$ | ½ | |
| | | $\therefore$ the root is in $(1.25,\ 1.5)$. | | |
| | | $\therefore x_3=\frac{1.25+1.5}{2}=1.375$ | ½ | **4** |
| | | **OR** | | |

<table>
<tr><th>Que. No.</th><th>Sub. Que.</th><th>Model Answers</th><th>Marks</th><th>Total Marks</th></tr>
<tr><td>5)</td><td></td><td>$x^2+x-3=0$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$f(x)=x^2+x-3$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\therefore f(1)=-1$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$f(2)=3$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$\therefore$ the root is in $(1,\ 2)$.</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>
<table>
<tr><th>a</th><th>b</th><th>$x=\frac{a+b}{2}$</th><th>$f(x)$</th></tr>
<tr><td>1</td><td>2</td><td>1.5</td><td>0.75</td></tr>
<tr><td>1</td><td>1.5</td><td>1.25</td><td>-0.188</td></tr>
<tr><td>1.25</td><td>1.5</td><td>1.375</td><td>---</td></tr>
</table>
</td><td>1<br>1<br>½</td><td>4</td></tr>
<tr><td></td><td></td><td>------------------------------------------------------------------------------------</td><td></td><td></td></tr>
<tr><td></td><td>d)</td><td>Using Newton-Raphson method find approximate value of $\sqrt[3]{100}$ (perform three iterations).</td><td></td><td></td></tr>
<tr><td></td><td>Ans.</td><td>$Let\ \ x=\sqrt[3]{100}$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\therefore x^3-100=0$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\therefore f(x)=x^3-100$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\therefore f'(x)=3x^2$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$\therefore f(4)=-36$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$f(5)=25$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$x-\frac{f(x)}{f'(x)}=x-\frac{x^3-100}{3x^2}$ ---(*)</td><td>1</td><td></td></tr>
<tr><td></td><td></td><td>$=\frac{x(3x^2)-(x^3-100)}{3x^2}$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$=\frac{2x^3+100}{3x^2}$ ---(**)</td><td></td><td></td></tr>
<tr><td></td><td></td><td>Start with $x_0=5$,</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\therefore x_1=4.667$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$x_2=4.642$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$x_3=4.642$</td><td>½</td><td>4</td></tr>
<tr><td></td><td></td><td>**Note i)** If the problem is solved by taking $f(x)=x-\sqrt[3]{100}$, no marks to be given since to find various values of $f(x)$ for different values of $x$, it is required to use the value of $\sqrt[3]{100}$ and it is not permissible in this example as here given task is to find its approximate value.</td><td></td><td></td></tr>
</table>

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| **5)** | | **Note ii)** Once the formula (*) is formed, writing the direct values of $x_i's$ is permissible, as we allow it in case of Table Format for either bisection method or regula-falsi method. | | |
| | | **Note iii)** To calculate directly the values of $x_i's$, students may use the formula (*) instead of formulating the reduced form (**) of (*). This is also considerable. No marks to be deducted. | | |
| | | **OR** | | |
| | | $\therefore f(x)=x^3-100$ | | |
| | | $\therefore f'(x)=3x^2$ | ½ | |
| | | $\therefore f(4)=-36$ | ½ | |
| | | $f(5)=25$ | ½ | |
| | | $\therefore$ the root is in (4, 5). | | |
| | | $\therefore$ start with $x_0=5$ | | |
| | | $\therefore x_1=x_0-\frac{f(x_0)}{f'(x_0)}$<br>$=5-\frac{f(5)}{f'(5)}$<br>$=5-\frac{25}{75}$<br>$=4.667$ | 1 | |
| | | $x_2=x_1-\frac{f(x_1)}{f'(x_1)}$<br>$=4.667-\frac{f(4.667)}{f'(4.667)}$<br>$=4.667-\frac{1.651}{65.343}$<br>$=4.642$ | 1 | |
| | | $x_3=x_2-\frac{f(x_2)}{f'(x_2)}$<br>$=4.642-\frac{f(4.642)}{f'(4.642)}$<br>$=4.642-\frac{0.027}{64.644}$<br>$=4.642$ | ½ | **4** |

<table>
<tr><th>Que. No.</th><th>Sub. Que.</th><th>Model Answers</th><th>Marks</th><th>Total Marks</th></tr>
<tr><td>**5)**</td><td>e)</td><td>Using regula-falsi method find approximate root of equation $x^3+2x^2-8=0$ (take three iterations).</td><td></td><td></td></tr>
<tr><td></td><td>Ans.</td><td>$x^3+2x^2-8=0$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$f(x)=x^3+2x^2-8$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\therefore f(1)=-5$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$f(2)=8$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$\therefore$ the root is in $(1,\ 2)$.</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$\therefore x_1=\dfrac{af(b)-bf(a)}{f(b)-f(a)}=\dfrac{1(8)-2(-5)}{8-(-5)}=1.385$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$\therefore f(1.385)=-1.507$</td><td>½</td><td></td></tr>
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<tr><td></td><td></td><td>$\therefore x_3=1.504$</td><td>½</td><td>**4**</td></tr>
<tr><td></td><td></td><td>**OR**</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$x^3+2x^2-8=0$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$f(x)=x^3+2x^2-8$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\therefore f(1)=-5$</td><td>½</td><td></td></tr>
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<tr><td></td><td>f)</td><td>Find the real root of the equation $x\cdot e^x=3$ using false position method (two iterations only).</td><td></td><td></td></tr>
<tr><td></td><td>Ans.</td><td>$x\cdot e^x=3$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>$\therefore f(x)=x\cdot e^x-3$</td><td></td><td></td></tr>
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<tr><td></td><td></td><td>$\therefore f(1) = -0.282$</td><td>½</td><td></td></tr>
<tr><td></td><td></td><td>$f(2) = 11.778$</td><td>½</td><td></td></tr>
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<tr><td></td><td></td><td>$\therefore x_1 = \dfrac{af(b)-bf(a)}{f(b)-f(a)} = \dfrac{1(11.778)-2(-0.282)}{11.778-(-0.282)} = 1.023$</td><td>1½</td><td></td></tr>
<tr><td></td><td></td><td>$\therefore f(1.023) = -0.154$</td><td></td><td></td></tr>
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<tr><td></td><td></td><td>$x \cdot e^x = 3$</td><td></td><td></td></tr>
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</td><td>1 ½<br>1 ½</td><td><b>4</b></td></tr>
<tr><td></td><td></td><td><b>Note for Numerical Problems:</b> For practical purpose, generally the values of fractional numbers are truncated up to 3 decimal points by the method of rounded-off. Thus the solution is taken up to 3 decimal points only. If answer is truncated more than 3 decimal points, the final answer may vary for last decimal point. Due to the use of advance calculators, such as modern scientific non-programmable calculators, 1/3 is actually 0.333333333333 but can be taken as 0.333 or in case of 3/7 it is actually 0.428571428 but it is truncated as 0.429. Further it is preferred that in numerical methods the answers are to be in decimal forms, but still many times students keep answers in fractional form. In this case, no marks to be deducted.<br>-----------------------------------------------------------------------------------</td><td></td><td></td></tr>
</table>

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| **6)** | | **Attempt any Four of the followings:** | | |
| | a) | Differentiate $\cos^{-1}\left(2x^2-1\right)$ w. r. t. $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$. | | |
| | Ans. | $Let\ \ u=\cos^{-1}\left(2x^2-1\right)$ | 1 | |
| | | $\therefore \cos u=2x^2-1$ | | |
| | | $\therefore \sin u=\sqrt{1-\cos^2 u}=\sqrt{1-\left(2x^2-1\right)^2}=\sqrt{-4x^4+4x^2}=2x\sqrt{1-x^2}$ | 1 | |
| | | $Let\ \ v=\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ | | |
| | | $\therefore v=\sin^{-1}\left(\sin u\right)=u$ | 1 | |
| | | $\therefore u=v$ | | |
| | | $\therefore \dfrac{du}{dv}=1$ | 1 | **4** |
| | | **OR** | | |
| | | $Let\ \ u=\cos^{-1}\left(2x^2-1\right)$ | | |
| | | $\therefore \dfrac{du}{dx}=\dfrac{-1}{\sqrt{1-\left(2x^2-1\right)^2}}\cdot\dfrac{d}{dx}\left(2x^2-1\right)$ | ½ | |
| | | $=\dfrac{-4x}{\sqrt{4x^2-4x^4}}$ | ½ | |
| | | $=\dfrac{-4x}{2x\sqrt{1-x^2}}$ | | |
| | | $=\dfrac{-2}{\sqrt{1-x^2}}$ | ½ | |
| | | $Let\ \ v=\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ | | |
| | | $\therefore \dfrac{dv}{dx}=\dfrac{1}{\sqrt{1-\left(2x\sqrt{1-x^2}\right)^2}}\cdot\dfrac{d}{dx}\left(2x\sqrt{1-x^2}\right)$ | ½ | |
| | | $=\dfrac{1}{\sqrt{1-\left[4x^2\left(1-x^2\right)\right]}}\cdot\left(2x\cdot\dfrac{1}{2\sqrt{1-x^2}}\cdot(-2x)+\sqrt{1-x^2}\cdot 2\right)$ | | |
| | | $=\dfrac{1}{\sqrt{1-4x^2+4x^4}}\cdot\left(\dfrac{-2x^2}{\sqrt{1-x^2}}+2\sqrt{1-x^2}\right)$ | | |
| | | $=\dfrac{1}{\sqrt{\left(1-2x^2\right)^2}}\cdot\left(\dfrac{-2x^2+2\left(1-x^2\right)}{\sqrt{1-x^2}}\right)$ | | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| **6)** | | $=\frac{2\left(1-2x^2\right)}{\left(1-2x^2\right)\sqrt{1-x^2}}$ | | |
| | | $=\frac{2}{\sqrt{1-x^2}}$ | 1 | |
| | | $\therefore \frac{du}{dv}=\frac{du/d\theta}{dv/d\theta}=\frac{\frac{-2}{\sqrt{1-x^2}}}{\frac{2}{\sqrt{1-x^2}}}=-1$ | 1 | **4** |
| | | **OR** | | |
| | | $Let\ \ u=\cos^{-1}\left(2x^2-1\right)$ | | |
| | | $Put\ x=\cos\theta$ | | |
| | | $\therefore u=\cos^{-1}\left(2\cos^2\theta-1\right)=\cos^{-1}\left(\cos 2\theta\right)=2\theta$ | | |
| | | $=2\cos^{-1}x$ | ½ | |
| | | $\therefore \frac{du}{d\theta}=\frac{-2}{\sqrt{1-x^2}}$ | 1 | |
| | | $Let\ \ v=\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ | | |
| | | $Put\ x=\sin\theta \qquad \left(or\ also\ x=\cos\theta\right)$ | | |
| | | $\therefore v=\sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$ | | |
| | | $=\sin^{-1}\left(2\sin\theta\sqrt{\cos^2\theta}\right)$ | | |
| | | $=\sin^{-1}\left(2\sin\theta\cos\theta\right)$ | | |
| | | $=\sin^{-1}\left(\sin 2\theta\right)$ | | |
| | | $=2\theta$ | | |
| | | $=2\sin^{-1}x \qquad \left(or\ also\ v=2\cos^{-1}x\right)$ | ½ | |
| | | $\therefore \frac{dv}{d\theta}=\frac{2}{\sqrt{1-x^2}} \qquad \left(or\ also\ \frac{dv}{d\theta}=\frac{-2}{\sqrt{1-x^2}}\right)$ | 1 | |
| | | $\therefore \frac{du}{dv}=\frac{du/d\theta}{dv/d\theta}=\frac{\frac{-2}{\sqrt{1-x^2}}}{\frac{2}{\sqrt{1-x^2}}}=-1 \qquad \left(or\ also\ \frac{du}{dv}=1\right)$ | 1 | **4** |
| | | ------------------------------------------------------------------------------------ | | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| **6)** | | | | |
| | b) | If $y=\sin 5x-3\cos 5x$, prove that $\frac{d^{2}y}{dx^{2}}+25y=0$. | | |
| | Ans. | $y=\sin 5x-3\cos 5x$ | | |
| | | $\therefore \frac{dy}{dx}=\cos 5x\cdot 5+3\sin 5x\cdot 5$<br>$=5\cos 5x+15\sin 5x$ | 1 | |
| | | $\therefore \frac{d^{2}y}{dx^{2}}=-25\sin 5x+75\cos 5x$ | 1 | |
| | | $=-25\left(\sin 5x-3\cos 5x\right)$<br>$=-25y$ | 1 | |
| | | $\therefore \frac{d^{2}y}{dx^{2}}+25y=0$ | 1 | **4** |
| | | **OR** | | |
| | | $y=\sin 5x-3\cos 5x$ | | |
| | | $\therefore \frac{dy}{dx}=5\cos 5x+15\sin 5x$ | 1 | |
| | | $\therefore \frac{d^{2}y}{dx^{2}}=-25\sin 5x+75\cos 5x$ | 1 | |
| | | $\therefore \frac{d^{2}y}{dx^{2}}+25y=-25\sin 5x+75\cos 5x+25\left(\sin 5x-3\cos 5x\right)$ | 1 | |
| | | $=-25\sin 5x+75\cos 5x+25\sin 5x-75\cos 5x$<br>$=0$ | 1 | **4** |
| | | ------------------------------------------------------------------------------------ | | |
| | c) | Solve the following equations by Jacobi's method, performing three iterations only:<br>$20x+y-2z=17,\quad 3x+20y-z=-18,\quad 2x-3y+20z=25$ | | |
| | Ans. | $20x+y-2z=17$<br>$3x+20y-z=-18$<br>$2x-3y+20z=25$ | | |
| | | $\therefore x=\frac{1}{20}\left(17-y+2z\right)$<br>$y=\frac{1}{20}\left(-18-3x+z\right)$<br>$z=\frac{1}{20}\left(25-2x+3y\right)$ | 1 | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| **6)** | | Starting with $x_0 = 0 = y_0 = z_0$<br>$x_1 = 0.85$<br>$y_1 = -0.9$<br>$z_1 = 1.25$ | 1 | |
| | | $x_2 = 1.02$<br>$y_2 = -0.965$<br>$z_2 = 1.03$ | 1 | |
| | | $x_3 = 1.001$<br>$y_3 = -1.002$<br>$z_3 = 1.003$ | 1 | **4** |
| | | ------------------------------------------------------------------------------------- | | |
| | d) | Solve the following equations by Gauss-Seidal method, taking three iterations only:<br>$15x + 2y + z = 18, \quad 2x + 20y - 3z = 19, \quad 3x - 6y + 25z = 22$ | | |
| | Ans. | $\therefore x = \frac{1}{15}(18 - 2y - z)$<br>$y = \frac{1}{20}(19 - 2x + 3z)$<br>$z = \frac{1}{25}(22 - 3x + 6y)$ | 1 | |
| | | Starting with $x_0 = 0 = y_0 = z_0$<br>$x_1 = 1.2$<br>$y_1 = 0.83$<br>$z_1 = 0.935$ | 1 | |
| | | $x_2 = 1.027$<br>$y_2 = 0.988$<br>$z_2 = 0.994$ | 1 | |
| | | $x_3 = 1.002$<br>$y_3 = 0.999$<br>$z_3 = 0.999$ | 1 | **4** |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| **6)** | | | | |
| | e) | Solve the following equations by Gauss elimination method:<br>$x+2y+3z=14, \quad 3x+y+2z=11, \quad 2x+3y+z=11$ | | |
| | Ans. | $x+2y+3z=14$<br>$3x+y+2z=11$<br>$2x+3y+z=11$ | | |
| | | $\begin{array}{l} 3x+6y+9z=42 \\ 3x+y+2z=11 \\ \underline{-\quad -\quad -\qquad -} \\ \quad 5y+7z=31 \end{array}$ and $\begin{array}{l} 6x+2y+4z=22 \\ 6x+9y+3z=33 \\ \underline{-\quad -\quad -\quad -} \\ \quad -7y+z=-11 \end{array}$ | 1 | |
| | | $\begin{array}{l} \quad 5y+7z=31 \\ -49y+7z=-77 \\ \underline{+\qquad -\qquad +} \\ 54y=108 \end{array}$ | | |
| | | $\therefore y=2$ | 1 | |
| | | $z=3$ | 1 | |
| | | $x=1$ | 1 | **4** |
| | | **OR** | | |
| | | $x+2y+3z=14$<br>$3x+y+2z=11$<br>$2x+3y+z=11$ | | |
| | | $\begin{array}{l} x+2y+3z=14 \\ 6x+2y+4z=22 \\ \underline{-\quad -\quad -\qquad -} \\ \quad -5x-z=-8 \end{array}$ and $\begin{array}{l} 9x+3y+6z=33 \\ 2x+3y+z=11 \\ \underline{-\quad -\quad -\quad -} \\ 7x+5z=22 \end{array}$ | 1 | |
| | | $\begin{array}{l} -25x-5z=-40 \\ \underline{\quad 7x+5z=22} \\ -18x=-18 \end{array}$ | | |
| | | $\therefore x=1$ | 1 | |
| | | $z=3$ | 1 | **4** |
| | | $y=2$ | 1 | |
| | | **OR** | | |
| | | $x+2y+3z=14$<br>$3x+y+2z=11$<br>$2x+3y+z=11$ | | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| **6)** | | $\begin{array}{l} 2x+4y+6z=28\\ 9x+3y+6z=33\\ \underline{-\quad -\quad -\quad\quad -}\\ -7x+y=-5\end{array}$ and $\begin{array}{l} 3x+y+2z=11\\ 4x+6y+2z=22\\ \underline{-\quad -\quad\quad -\quad -}\\ -x-5y=-11\end{array}$ | 1 | |
| | | $\begin{array}{l} -35x+5y=-25\\ \underline{-x-5y=-11}\\ -36x=-36\end{array}$ | | |
| | | $\therefore x=1$<br>$y=2$<br>$z=3$ | 1<br>1<br>1 | **4** |
| | | **Note:** In the method I, first x is eliminated and then z is eliminated to find the value of y first.<br>Whereas in the method II, first y is eliminated and then z is eliminated to find the value of x first.<br>Similarly in the method III, first z is eliminated and then y is eliminated to find the value of x first.<br>These are just illustrations to get desire solution. But student may follow another order of solution just on this line of solution i. e., to say in the method I, student may first eliminate x and then y to find the value of z first, appropriate marks to be given as per above scheme of marking. | | |
| | | ------------------------------------------------------------------------------------- | | |
| | f) | With the following system of equations: $5x-y=9,\quad x-5y+z=-4,\quad y-5z=6$ set up the Gauss-Seidal iterations scheme for solution. Iterate two times, using initial approximations as $x_0=1.5,\quad y_0=0.5,\quad z_0=-0.5$ | | |
| | Ans. | $\begin{array}{l} 5x-y=9\\ x-5y+z=-4\\ y-5z=6\end{array}$ OR $\begin{array}{l} 5x-y+0z=9\\ x-5y+z=-4\\ 0x+y-5z=6\end{array}$<br>**Note :** Student may use any one of these system for further solution. | | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| **6)** | | $\therefore x=\frac{1}{5}(9+y)$ | | |
| | | $y=\frac{1}{-5}(-4-x-z)$ | 1 | |
| | | $z=\frac{1}{-5}(6-y)$ | | |
| | | Starting with $x_0=1.5,\quad y_0=0.5,\quad z_0=-0.5$ | | |
| | | $x_1=1.9$ | ½ | |
| | | $y_1=1.08$ | ½ | |
| | | $z_1=-0.984$ | ½ | |
| | | $x_2=2.016$ | ½ | |
| | | $y_2=1.006$ | ½ | |
| | | $z_2=-0.999$ | ½ | **4** |
| | | ---------------------------------------- | | |
| | | ----------------------------------------<br>---------------------------------------- | | |
| | | **Important Note** | | |
| | | In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. **In such case, FIRST SEE whether the method falls within the scope of the curriculum**, and THEN ONLY give appropriate marks in accordance with the scheme of marking. | | |
| | | ----------------------------------------<br>---------------------------------------- | | |